

A Compromise between Neutrino Masses and Collider Signatures in the Type-II Seesaw Model

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Abstract

A natural extension of the standard $SU(2)_L \times U(1)_Y$ gauge model to accommodate massive neutrinos is to introduce one Higgs triplet and three right-handed Majorana neutrinos, leading to a 6×6 neutrino mass matrix which contains three 3×3 sub-matrices M_L , M_D and M_R . We show that three light Majorana neutrinos (i.e., the mass eigenstates of ν_e , ν_μ and ν_τ) are exactly massless in this model, if and only if $M_L = M_D M_R^{-1} M_D^T$ exactly holds. This no-go theorem implies that small but non-vanishing neutrino masses may result from a significant but incomplete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms in the Type-II seesaw formula, provided three right-handed Majorana neutrinos are of $\mathcal{O}(1)$ TeV and experimentally detectable at the LHC. We propose three simple Type-II seesaw scenarios with the $A_4 \times U(1)_X$ flavor symmetry to interpret the observed neutrino mass spectrum and neutrino mixing pattern. Such a TeV-scale neutrino model can be tested in two complementary ways: (1) searching for possible collider signatures of lepton number violation induced by the right-handed Majorana neutrinos and doubly-charged Higgs particles; and (2) searching for possible consequences of unitarity violation of the 3×3 neutrino mixing matrix in the future long-baseline neutrino oscillation experiments.

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I. INTRODUCTION

The solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have provided us with very convincing evidence that neutrinos are massive and lepton flavors are mixed. This important discovery indicates that the Standard Model (SM), in which neutrinos are massless and lepton flavors are conserved, is actually incomplete. In order to generate tiny neutrino masses, one may naturally extend the SM by introducing three right-handed Majorana neutrinos and one Higgs triplet but preserving the $SU(2)_L \times U(1)_Y$ gauge symmetry. The relevant Lagrangian for lepton masses can be written as

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l \tilde{H} E_R + \bar{l}_L Y_\nu N_R H + \frac{1}{2} \bar{N}_R^c M_R N_R + \frac{1}{2} Y_\Delta \bar{l}_L i\sigma_2 \Delta_L l_L^c + \text{h.c.} , \quad (1)$$

where l_L is the lepton doublet, H with $\tilde{H} \equiv i\sigma_2 H^*$ is the Higgs doublet, E_R and N_R stand respectively for the $SU(2)_L$ singlets of charged leptons and neutrinos, and Δ_L denotes the Higgs triplet. After spontaneous symmetry breaking, we obtain the mass matrices $M_l = Y_l v$, $M_L = Y_\Delta v_L$ and $M_D = Y_\nu v$, where $v = \langle H^0 \rangle$ and $v_L = \langle \Delta_L \rangle$ are the vacuum expectation values (vev's) of the neutral components of scalar fields H and Δ_L , respectively. A precision measurement of the ρ -parameter [5] strictly constrains the tree-level contribution of the Higgs triplet to the SM, and thus we are left with $v_L \lesssim 1$ GeV together with $v \approx 174$ GeV. The mass scale of M_R , which is not subject to the gauge symmetry breaking scale, can be much higher than v . To the leading order, the effective mass matrix for three light neutrinos is determined by the Type-II seesaw formula $M_\nu \approx M_L - M_D M_R^{-1} M_D^T$ [6, 7]. If the Higgs triplet Δ_L is absent, the small mass scale of M_ν can be just attributed to the large mass scale of M_R (i.e., the Type-I seesaw mechanism [6]). In the absence of heavy right-handed Majorana neutrinos, the observed smallness of three neutrino masses implies that the mass scale of M_L should be extremely small. A general case is that both terms of M_ν are important (e.g., comparable in magnitude) and their significant cancellation leads to small neutrino masses. In connection with the origin of neutrino masses, the phenomenon of lepton flavor mixing arises from the mismatch between the diagonalizations of M_l and M_ν .

Seesaw mechanisms are currently the most natural way to generate tiny neutrino masses, and they can naturally be embedded into more fundamental frameworks such as the grand unified theories (GUT's) or string theory. Typical examples of this nature are the $SO(10)$ GUT's [8] and the $E_8 \times E_8$ heterotic string theory [9]. A salient feature of most seesaw models is that the thermal leptogenesis mechanism [10] can work well to account for the

cosmological baryon number asymmetry via the CP-violating and out-of-equilibrium decays of heavy right-handed neutrinos and the $(B - L)$ -conserving sphaleron processes. On the experimental side, however, how to test seesaw mechanisms has been a question. Given the light neutrino mass scale $m_\nu \sim 0.01$ eV and $Y_\nu \sim \mathcal{O}(1)$ in the Type-I seesaw scenario, the mass scale of right-handed Majorana neutrinos is expected to be $m_R \sim 10^{15}$ GeV as a straightforward consequence of the inverted seesaw formula $M_R \approx -M_D^T M_\nu^{-1} M_D$. Such neutral particles can never be produced and detected at any colliders even in the far future, not only because they are too heavy but also because the strength of their charged-current interactions (characterized by the ratio $M_D M_R^{-1} \sim \sqrt{m_\nu/m_R} \sim 10^{-13}$) is too small. A possible way out is to lower the mass scale of M_R down to the TeV level but allow the Yukawa coupling matrix Y_ν to be of $\mathcal{O}(10^{-3})$ up to $\mathcal{O}(1)$. In order to generate sufficiently small neutrino masses in this kind of TeV-scale seesaw scenarios [11, 12], the key point is to adjust the textures of M_D and M_R to guarantee $M_D M_R^{-1} M_D^T = 0$ in the leading-order approximation. Then tiny but non-vanishing neutrino masses can be ascribed to slight perturbations or radiative corrections to $M_D M_R^{-1} M_D^T$ in the next-to-leading order approximation. Although such a seesaw model seems quite contrived, it is hopeful to be tested at the Large Hadron Collider (LHC) by searching for clear lepton-number-violating signals induced by heavy Majorana neutrinos [13]. Recently, Kersten and Smirnov [14] have reconsidered this sort of *structural cancellation* in the Type-I seesaw formula and pointed out some possible flavor symmetries behind it. One of their important observations is that the main structures of M_D and M_R , which are relevant to possibly observable collider signatures, are difficult to imprint on those sub-leading effects (due to explicit perturbations or radiative corrections) responsible for tiny neutrino masses. In other words, collider physics seems to be essentially decoupled from neutrino physics in generic Type-I seesaw scenarios [14], no matter whether the heavy Majorana neutrinos are of $\mathcal{O}(1)$ TeV or much heavier than that.

This work aims to extend Kersten and Smirnov's consideration to the Type-II seesaw case with both the right-handed Majorana neutrinos and the Higgs triplet at the TeV scale. This extension is non-trivial and intriguing at least in two aspects: (a) instead of realizing the structural cancellation (i.e., $M_D M_R^{-1} M_D^T \approx 0$), we consider the *global cancellation* between the contribution from Δ_L and that from right-handed Majorana neutrinos (i.e., $M_L - M_D M_R^{-1} M_D^T \approx 0$); (b) not only the TeV-scale Majorana neutrinos but also the doubly-charged components of Δ_L are possible to show up in the collider experiments. In fact, the

long-lived doubly-charged scalar has already been searched for at the Tevatron [15]. We shall prove a no-go theorem: the masses of light Majorana neutrinos are exactly vanishing at the tree level if and only if the global cancellation $M_L - M_D M_R^{-1} M_D^T = 0$ exactly holds in generic Type-II seesaw scenarios. Therefore, a feasible way to obtain both tiny neutrino masses and appreciable collider signatures is to allow for an incomplete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms. To be explicit, we shall propose three simple type-II seesaw scenarios with the $A_4 \times U(1)_X$ flavor symmetry at the TeV scale, from which the observed neutrino mass spectrum and neutrino mixing pattern can be achieved. We shall also discuss two interesting consequences of this model: (1) possible unitarity violation of the 3×3 neutrino mixing matrix, which can be searched for in the future long-baseline neutrino oscillation experiments; and (2) possible signatures of lepton number violation induced by the right-handed Majorana neutrinos and doubly-charged Higgs particles, which can be searched for at the LHC and other colliders.

The remaining part of this paper is organized as follows. In section II, we review some basics of the type-II seesaw mechanism and prove the no-go theorem. Section III is devoted to a specific type-II seesaw model, in which the incomplete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms is realized by the $A_4 \times U(1)_X$ symmetry and its breaking. The unitarity violation of the 3×3 neutrino mixing matrix and possible collider signatures of lepton number violation are discussed in section IV. Some conclusions are drawn in section V.

II. TYPE-II SEESAW AND NO-GO THEOREM

We regularize our notations and conventions in this section by reviewing some basics of the Type-II seesaw mechanism. After spontaneous symmetry breaking, the lepton mass terms in Eq. (1) turn out to be

$$-\mathcal{L}_{\text{mass}} = \overline{E_L} M_l E_R + \frac{1}{2} \overline{(\nu_L \ N_R^c)} \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.} , \quad (2)$$

where $\nu_L^c \equiv C \overline{\nu_L}^T$ with C being the charge conjugation matrix, likewise for N_R^c . The overall 6×6 neutrino mass matrix in $\mathcal{L}_{\text{mass}}$, denoted as \mathcal{M} , can be diagonalized by the unitary transformation $\mathcal{U}^\dagger \mathcal{M} \mathcal{U}^* = \widehat{\mathcal{M}}$; or explicitly,

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix} , \quad (3)$$

where $\widehat{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ and $\widehat{M}_N = \text{Diag}\{M_1, M_2, M_3\}$ with m_i and M_i (for $i = 1, 2, 3$) being the light and heavy Majorana neutrino masses, respectively. Note that the 3×3 rotation matrices V , U , R and S are non-unitary, but they are correlated with one another due to the unitarity of \mathcal{U} :

$$\begin{aligned} V^\dagger V + S^\dagger S &= VV^\dagger + RR^\dagger = \mathbf{1} , \\ U^\dagger U + R^\dagger R &= UU^\dagger + SS^\dagger = \mathbf{1} , \end{aligned} \quad (4a)$$

and

$$R^\dagger V + U^\dagger S = SV^\dagger + UR^\dagger = \mathbf{0} . \quad (4b)$$

The effective neutrino mass matrix M_ν can be defined by decomposing \mathcal{U} into a product of two unitary matrices \mathcal{W} and \mathcal{V} :

$$\mathcal{V}^\dagger \mathcal{W}^\dagger \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \mathcal{W}^* \mathcal{V}^* \equiv \mathcal{V}^\dagger \begin{pmatrix} M_\nu & \mathbf{0} \\ \mathbf{0} & M_N \end{pmatrix} \mathcal{V}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix} , \quad (5)$$

where \mathcal{W} and \mathcal{V} take the general forms

$$\mathcal{W} = \begin{pmatrix} U_1 & B \\ C & U_2 \end{pmatrix} , \quad \mathcal{V} = \begin{pmatrix} V_1 & \mathbf{0} \\ \mathbf{0} & V_2 \end{pmatrix} . \quad (6)$$

The 3×3 rotation matrices U_1 , B , C and U_2 are non-unitary, but they satisfy the normalization and orthogonality conditions of \mathcal{W} just like the correlative conditions of V , R , S and U given in Eq. (4). In contrast, V_1 and V_2 are unitary. It is trivial to obtain the relations $V = U_1 V_1$ and $R = B V_2$ from $\mathcal{U} = \mathcal{W} \mathcal{V}$. To express M_ν as a recursive expansion in powers of $M_D M_R^{-1}$, an ansatz has been made for \mathcal{W} in Ref. [16], in which $C = -B^\dagger$, $U_1 = \sqrt{\mathbf{1} - B B^\dagger}$ and $U_2 = \sqrt{\mathbf{1} - B^\dagger B}$ are reasonably assumed. More general but less instructive expressions of M_ν and M_N can be found in Ref. [17]. To the leading order,

$$M_\nu \approx M_L - M_D M_R^{-1} M_D^T , \quad (7)$$

known as the Type-II seesaw formula.

After diagonalizing \mathcal{M} , one may express the neutrino flavor eigenstates ν_α (for $\alpha = e, \mu, \tau$) in terms of the light and heavy neutrino mass eigenstates ν_i and N_i (for $i = 1, 2, 3$):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L . \quad (8)$$

In the basis where the flavor eigenstates of three charged leptons are identified with their mass eigenstates, the standard charged-current interactions between ν_α and α (for $\alpha = e, \mu, \tau$) turn out to be

$$-\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} \left[\overline{(e \ \mu \ \tau)}_{\text{L}} V \gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\text{L}} W_\mu^- + \overline{(e \ \mu \ \tau)}_{\text{L}} R \gamma^\mu \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\text{L}} W_\mu^- \right] + \text{h.c.} \quad (9)$$

It becomes clear that V describes the charged-current interactions of three light Majorana neutrinos (ν_1, ν_2, ν_3), while R is relevant to the charged-current interactions of three heavy Majorana neutrinos (N_1, N_2, N_3). One may similarly write out the interactions between the Majorana neutrinos and the neutral gauge boson (or Higgs) in the chosen flavor basis [11]. It is mainly the strength of charged-current interactions that determines the production and detection probabilities of heavy Majorana neutrinos at hadron or e^+e^- colliders. To experimentally test a seesaw mechanism, two prerequisites have to be satisfied: the mass scale of N_i should be low enough and the magnitude of R should be large enough. But both of them may in general give rise to unacceptably sizable masses of ν_i through the seesaw formula. One possible way to get around this difficulty in the Type-II seesaw mechanism might be to dictate a complete cancellation between the leading terms M_{L} and $M_{\text{D}} M_{\text{R}}^{-1} M_{\text{D}}^T$ and generate tiny neutrino masses via the sub-leading terms of M_ν in Eq. (7). Such an idea is seemingly reasonable, but it does not work because of the following no-go theorem:

If and only if the relationship $M_{\text{L}} = M_{\text{D}} M_{\text{R}}^{-1} M_{\text{D}}^T$ is exactly satisfied in generic Type-II seesaw models, then three light Majorana neutrinos must be exactly massless.

In other words, imposing the pre-condition $M_{\text{L}} = M_{\text{D}} M_{\text{R}}^{-1} M_{\text{D}}^T$ on the 6×6 neutrino mass matrix \mathcal{M} will automatically guarantee $M_\nu = \widehat{M}_\nu = \mathbf{0}$ for three light Majorana neutrinos. Hence tiny neutrino masses can only be generated from an incomplete cancellation between M_{L} and $M_{\text{D}} M_{\text{R}}^{-1} M_{\text{D}}^T$ terms or from radiative corrections. A similar theorem is valid for the canonical seesaw mechanism by setting $M_{\text{L}} = 0$; i.e., three light Majorana neutrinos must be massless if and only if $M_{\text{D}} M_{\text{R}}^{-1} M_{\text{D}}^T = 0$ exactly holds in generic Type-I seesaw models.

Now let us prove the above theorem in a way without loss of any generality. Rewriting Eq. (3) as $\mathcal{M} \mathcal{U}^* = \mathcal{U} \widehat{\mathcal{M}}$ and doing the matrix multiplication on both left- and right-hand sides, we obtain

$$V \widehat{M}_\nu = M_{\text{L}} V^* + M_{\text{D}} S^*, \quad (10a)$$

$$S \widehat{M}_\nu = M_{\text{D}}^T V^* + M_{\text{R}} S^*, \quad (10b)$$

$$R\widehat{M}_N = M_L R^* + M_D U^* , \quad (10c)$$

$$U\widehat{M}_N = M_D^T R^* + M_R U^* . \quad (10d)$$

The first step of our proof is to derive $\widehat{M}_\nu = 0$ from the pre-condition $M_L = M_D M_R^{-1} M_D^T$. Multiplying Eq. (10b) by $M_D M_R^{-1}$ on the left and taking account of Eq. (10a) and $M_L = M_D M_R^{-1} M_D^T$, we get

$$(M_D M_R^{-1} S - V) \widehat{M}_\nu = \mathbf{0} . \quad (11)$$

Multiplying Eq. (10d) by $M_D M_R^{-1}$ on the left and taking account of Eq. (10c), we analogously arrive at

$$(M_D M_R^{-1} U - R) \widehat{M}_N = \mathbf{0} . \quad (12)$$

By definition, \widehat{M}_N is a diagonal matrix containing three real and positive eigenvalues (i.e., the masses of three heavy Majorana neutrinos). Hence the unique solution to Eq. (12) is $R = M_D M_R^{-1} U$. This result, together with $SV^\dagger + UR^\dagger = 0$ given in Eq. (4b), leads to

$$M_D M_R^{-1} S V^\dagger + R R^\dagger = \mathbf{0} . \quad (13)$$

Combining Eqs. (4a) and (13), we are then left with

$$(M_D M_R^{-1} S - V) V^\dagger = -\mathbf{1} . \quad (14)$$

The unit matrix on the right-hand side of Eq. (14) implies that the ranks of $(M_D M_R^{-1} S - V)$ and V^\dagger must be three, and thus the rank of \widehat{M}_ν must be zero as required by Eq. (11). Namely, $\widehat{M}_\nu = 0$ is an unavoidable consequence of $M_L = M_D M_R^{-1} M_D^T$. The second step of our proof is to show that $M_L = M_D M_R^{-1} M_D^T$ will hold if three light Majorana neutrinos are massless (i.e., $\widehat{M}_\nu = 0$). For this purpose, we rewrite Eq. (3) as $\mathcal{M} = \mathcal{U} \widehat{\mathcal{M}} \mathcal{U}^T$ and then impose $\widehat{M}_\nu = 0$ on it. Three sub-matrices of \mathcal{M} turn out to be

$$M_L = R \widehat{M}_N R^T , \quad M_R = U \widehat{M}_N U^T , \quad M_D = R \widehat{M}_N U^T . \quad (15)$$

It is easy to verify that $M_L = M_D M_R^{-1} M_D^T$ holds in consequence of Eq. (15), or equivalently in consequence of $\widehat{M}_\nu = 0$. This completes the proof of our theorem.

The no-go theorem tells us that it is impossible to generate tiny neutrino masses from the sub-leading seesaw terms in a recursive expansion of M_ν (in powers of $M_D M_R^{-1}$), if and only

if the condition $M_L = M_D M_R^{-1} M_D^T$ is imposed. This point has more or less been observed or illustrated in the literature (see, e.g., Refs. [11, 14, 16, 18]), but only our present work provides the most general proof without any special assumption or approximation. In order to reach a compromise between tiny neutrino masses and accessible collider signatures at the TeV scale, a phenomenologically viable way is to consider significant but incomplete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms in the Type-II seesaw formula. We shall propose a specific model with the $A_4 \times U(1)_X$ flavor symmetry to realize the desired cancellation in section III and discuss its consequences on collider physics in section IV.

III. A SPECIFIC MODEL WITH $A_4 \times U(1)_X$ SYMMETRY

To simultaneously achieve tiny neutrino masses and large neutrino mixing angles, we impose the $A_4 \times U(1)_X$ flavor symmetry [19] on the Type-II seesaw Lagrangian in Eq. (1). In this case, the assignments of relevant lepton and scalar fields with respect to the symmetry group $SU(2)_L \times U(1)_Y \otimes A_4 \times U(1)_X$ are

$$\begin{aligned}
l_L &\sim (2, -1) \otimes (\underline{3}, 1), & \phi &\sim (2, -1) \otimes (\underline{1}, 1), \\
E_R &\sim (1, -2) \otimes (\underline{1}, 1), & \Phi &\sim (2, -1) \otimes (\underline{3}, 0), \\
E'_R &\sim (1, -2) \otimes (\underline{1}', 1), & \chi &\sim (1, 0) \otimes (\underline{3}, 1), \\
E''_R &\sim (1, -2) \otimes (\underline{1}'', 1), & \Delta &\sim (3, -2) \otimes (\underline{1}, 2), \\
N_R &\sim (1, 0) \otimes (\underline{3}, 0), & \Sigma &\sim (3, -2) \otimes (\underline{3}, 0),
\end{aligned} \tag{16}$$

where several triplet scalars have been introduced. The irreducible representations of A_4 group and the decomposition of their direct products can be found in Ref. [20]. Given $SU(2)_L \times U(1)_Y \otimes A_4 \times U(1)_X$ invariance, the Lagrangian responsible for lepton masses reads

$$\begin{aligned}
-\mathcal{L}_{\text{lepton}} &= y_e \left(\overline{l}_L \tilde{\Phi} \right)_{\underline{1}} E_R + y'_e \left(\overline{l}_L \tilde{\Phi} \right)_{\underline{1}'} E''_R + y''_e \left(\overline{l}_L \tilde{\Phi} \right)_{\underline{1}''} E'_R \\
&\quad + \frac{1}{2} y_\Delta \overline{l}_L i \sigma_2 \Delta l_L^c + \frac{1}{2} m_R \left(\overline{N}_R^c N_R \right)_{\underline{1}} + y_\nu \left(\overline{l}_L N_R \right)_{\underline{1}} \phi + \text{h.c.},
\end{aligned} \tag{17}$$

in which the gauge-invariant and A_4 -invariant terms $\overline{l}_L N_R \Phi$, $\overline{N}_R^c N_R \chi$ and $\overline{l}_L i \sigma_2 \Sigma l_L^c$ do not appear because they are forbidden by the $U(1)_X$ symmetry. After spontaneous symmetry breaking, the overall neutrino mass matrix \mathcal{M} is determined by its three 3×3 sub-matrices

$$M_L = m_L \cdot \mathbf{1}, \quad M_D = m_D \cdot \mathbf{1}, \quad M_R = m_R \cdot \mathbf{1}, \tag{18}$$

where $m_L = y_\Delta \langle \Delta \rangle$ and $m_D = y_\nu \langle \phi \rangle$. In the assumption of $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \langle \Phi_3 \rangle$, the charged-lepton mass matrix can be written as $M_l = U_l \widehat{M}_l$, where $\widehat{M}_l = \text{Diag}\{m_e, m_\mu, m_\tau\} = \sqrt{3} \langle \Phi_i \rangle \text{Diag}\{y_e, y'_e, y''_e\}$ and

$$U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \quad (19)$$

with $\omega = \exp(i2\pi/3)$. It is quite obvious that $m_L = m_D^2/m_R$ will lead to $M_L = M_D M_R^{-1} M_D^T$. According to the no-go theorem, this complete cancellation makes light neutrino masses exactly vanishing. In order to obtain the realistic neutrino mass spectrum and lepton flavor mixing pattern, we may introduce an incomplete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms by breaking the flavor symmetry $U(1)_X$ explicitly to Z_2 . The $U(1)_X$ -violating terms, such as $(\Phi^\dagger \phi)_{\underline{3}} \cdot (\Phi^\dagger \phi)_{\underline{3}}$ in the scalar potential [20], can accomplish this purpose. For simplicity, we list the complete scalar potential in Appendix A. Note that the explicit breaking of the global $U(1)_X$ symmetry does not yield the problematic Goldstone particle. We may assign the proper Z_2 parity to produce slight perturbations to the neutrino mass terms. Three possibilities are discussed in order.

(1) Perturbations to M_L : l_L , E_R , E'_R , E''_R , χ and ϕ are odd under the Z_2 transformation, while the other fields are even under the same transformation. In this case, the Yukawa interaction $y_\Sigma \bar{l}_L i \sigma_2 \Sigma l_L^c$ is no longer forbidden and it contributes a few off-diagonal terms to the effective neutrino mass matrix:

$$M_\nu = \delta m \cdot \mathbf{1} + \begin{pmatrix} \mathbf{0} & \omega_3 & \omega_2 \\ \omega_3 & \mathbf{0} & \omega_1 \\ \omega_2 & \omega_1 & \mathbf{0} \end{pmatrix}, \quad (20)$$

where $\delta m = m_L - m_D^2/m_R$ is the residue of the incomplete cancellation induced by the mass terms in Eq. (18), and $\omega_i = y_\Sigma \langle \Sigma_i \rangle$ (for $i = 1, 2, 3$). In the assumption of $\langle \Sigma_1 \rangle = \langle \Sigma_3 \rangle = 0$ and $\langle \Sigma_2 \rangle \neq 0$, we get a more special texture of M_ν which can be diagonalized by the orthogonal transformation

$$V_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathbf{0} & -1 \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ 1 & \mathbf{0} & 1 \end{pmatrix}. \quad (21)$$

The mass eigenvalues of M_ν turn out to be $m_1 = |\delta m + \omega_2|$, $m_2 = |\delta m|$ and $m_3 = |\delta m - \omega_2|$. To be more explicit, we take $\delta m > 0$. Since $m_1 < m_2$ is required by current neutrino

oscillation data, we can obtain the normal neutrino mass hierarchy by setting $\omega_2 < 0$. Then the ratio of two neutrino mass-squared differences is given by $\Delta m_{21}^2/\Delta m_{32}^2 = (1-\alpha)/(1+\alpha)$ with $\alpha = |\omega_2|/2\delta m$. Taking $\Delta m_{21}^2 \approx 8.0 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 \approx 2.5 \times 10^{-3} \text{ eV}^2$ [21] as the typical inputs, we obtain $\alpha \approx 0.94$, $|\omega_2| \approx 0.035 \text{ eV}$ and $\delta m \approx 0.019 \text{ eV}$.

The lepton flavor mixing matrix V describes the mismatch between the diagonalizations of M_l and M_ν and is given by $V = U_l^\dagger U_1 V_1$, where U_1 and V_1 have generally been defined in Eq. (6). Note that the small deviation of U_1 from the unit matrix characterizes the unitarity violation of V , while V_1 is unitary and its expression has been given in Eq. (21). In the approximation of $U_1 \approx \mathbf{1}$, V is just the tri-bimaximal mixing pattern [22] compatible with current experimental data:

$$V \approx U_l^\dagger U_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \mathbf{0} \\ -\frac{1}{\sqrt{6}}\omega^2 & \frac{1}{\sqrt{3}}\omega^2 & -\frac{1}{\sqrt{2}}e^{-i\pi/6} \\ -\frac{1}{\sqrt{6}}\omega & \frac{1}{\sqrt{3}}\omega & -\frac{1}{\sqrt{2}}e^{+i\pi/6} \end{pmatrix}. \quad (22)$$

Thus this Type-II seesaw scenario is viable to interpret the observed neutrino mass spectrum and neutrino mixing pattern. Appreciable collider signatures can be achieved by adjusting the ratio m_D/m_R , which is apparently independent of the parameters responsible for the masses of light neutrinos (i.e., δm and ω_2), since the strength of charged-current interactions of heavy Majorana neutrinos N_i is essentially described by $R \approx U_l^\dagger m_D/m_R$. More discussions about the unitarity violation of V and possible collider signatures of N_i will be given in section IV.

(2) Perturbations to M_D : l_L , χ , Σ and ϕ are odd under the Z_2 transformation, while the other fields are even under the same transformation. In this case, the Yukawa interaction $y_\Sigma \bar{l}_L i\sigma_2 \Sigma l_L^c$ is again forbidden, so is the term $y'_\nu \bar{l}_L N_R \Phi$. However, one can resort to new scalar doublets Φ' — their $A_4 \times U(1)_X$ charges are the same as Φ 's but their Z_2 charge is opposite to Φ 's. Then the mass matrix M_D takes the form

$$M_D = m_D \begin{pmatrix} 1 & \mathbf{0} & \lambda \\ \mathbf{0} & 1 & \mathbf{0} \\ \lambda & \mathbf{0} & 1 \end{pmatrix}, \quad (23)$$

where $\lambda = y'_\nu \langle \Phi'_2 \rangle / m_D$ and $\langle \Phi'_1 \rangle = \langle \Phi'_3 \rangle = 0$, but the mass matrices M_L and M_R keep unchanged (i.e., $M_L = m_L \cdot \mathbf{1}$ and $M_R = m_R \cdot \mathbf{1}$). Using the Type-II seesaw formula, we get

$$M_\nu = \delta m \cdot \mathbf{1} - \frac{m_D^2}{m_R} \begin{pmatrix} \lambda^2 & \mathbf{0} & 2\lambda \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 2\lambda & \mathbf{0} & \lambda^2 \end{pmatrix}. \quad (24)$$

This effective neutrino mass matrix can also be diagonalized by the orthogonal transformation given in Eq. (21). Its three eigenvalues are found to be $m_1 \approx |\delta m - 2\lambda m_D^2/m_R|$, $m_2 = \delta m$ and $m_3 \approx |\delta m + 2\lambda m_D^2/m_R|$, where the terms of $\mathcal{O}(\lambda^2)$ or smaller have been neglected. Taking $\Delta m_{21}^2 \approx 8.0 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 \approx 2.5 \times 10^{-3} \text{ eV}^2$ [21] as the typical inputs, we obtain $\delta m \approx 0.019 \text{ eV}$ and $\lambda m_D^2/m_R \approx 0.018 \text{ eV}$. Given $m_R \sim 100 \text{ GeV}$ and $m_D/m_R \sim 0.1$ so as to make the heavy Majorana neutrinos detectable at the LHC, the magnitude of λ turns out to be $\lambda \sim 10^{-11}$ in order to generate the correct magnitude of light neutrino masses. Namely, the smallness of m_i is attributed to the tiny perturbation parameter δm and the $U(1)_X$ symmetry breaking parameter λ .

In this Type-II seesaw scenario, the lepton flavor mixing matrix $V = U_l^\dagger U_1 V_1 \approx U_l^\dagger V_1$ is the same as that given in Eq. (22), where the small effects of unitarity violation have been neglected. The strength of charged-current interactions of heavy Majorana neutrinos can also approximate to $R \approx U_l^\dagger m_D/m_R$, because λ is vanishingly small.

(3) Perturbations to M_R : l_L , E_R , E'_R , E''_R , Σ and ϕ are odd under the Z_2 transformation, while the other fields are even under the same transformation. In this case, the Z_2 -conserving term $y_\chi \overline{N_R^c} N_R \chi$ exists. Then the right-handed Majorana neutrino mass matrix reads

$$M_R = m_R \begin{pmatrix} 1 & \mathbf{0} & \varrho \\ \mathbf{0} & 1 & \mathbf{0} \\ \varrho & \mathbf{0} & 1 \end{pmatrix}, \quad (25)$$

where $\varrho = y_\chi \langle \chi_2 \rangle / m_R$ and $\langle \chi_1 \rangle = \langle \chi_3 \rangle = 0$, but the mass matrices M_L and M_D keep unchanged (i.e., $M_L = m_L \cdot \mathbf{1}$ and $M_D = m_D \cdot \mathbf{1}$). From the Type-II seesaw formula, we obtain

$$M_\nu = \delta m - \frac{m_D^2}{m_R} \frac{\varrho}{1 - \varrho^2} \begin{pmatrix} \varrho & \mathbf{0} & -1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -1 & \mathbf{0} & \varrho \end{pmatrix}. \quad (26)$$

The orthogonal transformation in Eq. (21) can also be used to diagonalize the effective neutrino mass matrix in Eq. (26). After a straightforward calculation, we get $m_1 \approx |\delta m + \varrho m_D^2/m_R|$, $m_2 \approx \delta m$ and $m_3 \approx |\delta m - \varrho m_D^2/m_R|$, where the terms of $\mathcal{O}(\varrho^2)$ or smaller have been omitted. Then $\delta m \approx 0.019 \text{ eV}$ and $|\varrho| m_D^2/m_R \approx 0.035 \text{ eV}$ are obtained from the typical inputs $\Delta m_{21}^2 \approx 8.0 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 \approx 2.5 \times 10^{-3} \text{ eV}^2$ [21]. Given $m_R \sim 100 \text{ GeV}$ and $m_D/m_R \sim 0.1$ to make the heavy Majorana neutrinos detectable at the LHC, the sign and magnitude of ϱ are required to be $\varrho < 0$ and $|\varrho| \sim 10^{-11}$ by current neutrino oscillation data.

In this scenario, the lepton flavor mixing matrix $V = U_l^\dagger U_1 V_1 \approx U_l^\dagger V_1$ is the same as that given in Eq. (22), where the small effects of unitarity violation have been neglected. The strength of charged-current interactions of heavy Majorana neutrinos can also approximate to $R \approx U_l^\dagger m_D/m_R$, due to the smallness of ϱ .

The scenarios given above illustrate three simple ways to deform the complete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms such that tiny neutrino masses can be generated through the Type-II seesaw formula. A general approach should include the perturbations to M_L , M_D and M_R together. Let us denote $M_{L,D,R}$ as a sum of the “symmetry” term and the “perturbation” term: $M_{L,D,R} = \tilde{M}_{L,D,R} + \delta M_{L,D,R}$. The residue of the incomplete cancellation between \tilde{M}_L and $\tilde{M}_D \tilde{M}_R^{-1} \tilde{M}_D^T$ terms is denoted by δM (i.e., $\delta M = \tilde{M}_L - \tilde{M}_D \tilde{M}_R^{-1} \tilde{M}_D^T$). Then the Type-II seesaw formula $M_\nu \approx M_L - M_D M_R^{-1} M_D^T$ can be re-expressed as

$$M_\nu \approx \delta M + \delta M_L + \tilde{M}_D \tilde{M}_R^{-1} \delta M_R \tilde{M}_R^{-1} \tilde{M}_D^T - \tilde{M}_D \tilde{M}_R^{-1} (\delta M_D)^T - \delta M_D \tilde{M}_R^{-1} \tilde{M}_D^T \quad (27)$$

to the first order of $\delta M_{L,D,R}$. It is easy to see that Eqs. (20), (24) and (26) are just the special cases of Eq. (27).

We have shown that it is possible to achieve a phenomenological compromise between tiny neutrino masses and accessible collider signatures in the Type-II seesaw scenarios with spontaneous and explicit breaking of the $A_4 \times U(1)_X$ flavor symmetry. Proper A_4 symmetry breaking is also necessary in the quark sector to account for the observed quark mass spectra and flavor mixing parameters, as discussed in Ref. [20]. Note that radiative corrections to the light neutrino masses may be very large due to the largeness of Yukawa interactions in a certain Type-I or Type-II seesaw model, but some detailed calculations have shown that these corrections are vanishing (or vanishingly small) in the limit of degenerate (or nearly degenerate) heavy Majorana neutrino masses [23]. This is just the case for three simple Type-II seesaw scenarios discussed above. On the other hand, the seesaw threshold effects are also negligible in our examples because of the (near) mass degeneracy of three heavy Majorana neutrinos.

IV. UNITARITY VIOLATION AND COLLIDER SIGNATURES

Now we proceed to discuss the unitarity violation and collider signatures in the Type-II seesaw model. The non-unitarity of the lepton flavor mixing matrix V is actually a common

feature of the seesaw models, as one can easily see from $VV^\dagger = \mathbf{1} - RR^\dagger \neq \mathbf{1}$ in Eq. (4a). Taking account of Eqs. (3), (5), (6) and (9), we may express V and R as $V = U_l^\dagger U_1 V_1$ and $R = U_l^\dagger B V_2$, where U_l is the unitary matrix defined to diagonalize the Hermitian matrix $M_l M_l^\dagger$ with M_l being the charge-lepton mass matrix. The 3×3 matrices U_1 , B , V_1 and V_2 can in principle be determined by the neutrino mass matrices M_L , M_D and M_R , and thus V and R should be calculable. In practice, one may resort to a recursive expansion of M_ν in powers of $M_D M_R^{-1}$ by taking the reasonable assumptions $C = -B^\dagger$, $U_1 = \sqrt{\mathbf{1} - BB^\dagger}$ and $U_2 = \sqrt{\mathbf{1} - B^\dagger B}$ [16]. Then $U_1 \approx \mathbf{1} - BB^\dagger/2$ and $B \approx U_l^\dagger M_D M_R^{-1}$ are two good approximations, from which

$$V \approx U_l^\dagger \left[\mathbf{1} - \frac{1}{2} U_l^\dagger M_D M_R^{-1} (M_R^{-1} M_D^T)^* U_l \right] V_1 \quad (28)$$

can be obtained. For simplicity, let us define $\xi \equiv U_l^{\dagger 2} M_D M_R^{-1} (M_R^{-1} M_D^T)^* U_l^2$. Note that the Hermitian matrix ξ is suppressed by two powers of $M_D M_R^{-1}$. Hence $V \approx U_l^\dagger V_1$ is unitary in the leading-order approximation [17]. To a better degree of accuracy, we have $V \approx (\mathbf{1} - \xi/2) U_l^\dagger V_1$ and $VV^\dagger \approx \mathbf{1} - \xi$. Then we arrive at $\xi \approx RR^\dagger$. Note also that ξ is in general complex and may give rise to some additional CP-violating effects in neutrino oscillations [24]. In the framework of two-flavor oscillations, where the non-trivial CP-violating phase of $U_l^\dagger V_1$ is negligible, it remains possible to get a CP-violating asymmetry between the probabilities of $\nu_\alpha \rightarrow \nu_\beta$ and $\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta$ transitions:

$$\frac{P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)}{P(\nu_\alpha \rightarrow \nu_\beta) + P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)} \propto |\xi_{\alpha\beta}| \sin \delta_{\alpha\beta} \quad (29)$$

with $\delta_{\alpha\beta} \equiv \arg(\xi_{\alpha\beta})$ for $\alpha, \beta = e, \mu, \tau$ [24]. When the specific Type-II seesaw scenarios proposed in section III are taken into account, we find $\xi \approx RR^\dagger \approx m_D^2/m_R^2 \cdot \mathbf{1}$ and thus $\delta_{\alpha\beta} \approx 0$. This result shows that there is almost no extra CP violation induced by the unitarity violation of V in our special examples. Nevertheless, the diagonal elements of ξ can be as large as $\mathcal{O}(10^{-2})$ for $m_D/m_R \sim \mathcal{O}(10^{-1})$, implying that the deviation of V from unitarity can actually reach the percent level. It is worth emphasizing that such a model-dependent argument has no conflict with the model-independent bound on VV^\dagger or equivalently on ξ . A global analysis of current neutrino oscillation data and precision electroweak data (e.g., on the invisible width of the Z^0 boson, universality tests and rare lepton decays) has yielded quite strong constraints on the unitarity of V and its possible violation [25]. Translating the numerical results of Refs. [24] and [25] into the restriction on

ξ in our language, we obtain

$$|\xi| = \begin{pmatrix} |\xi_{ee}| < 1.1 \cdot 10^{-2} & |\xi_{e\mu}| < 7.0 \cdot 10^{-5} & |\xi_{e\tau}| < 1.6 \cdot 10^{-2} \\ |\xi_{\mu e}| < 7.0 \cdot 10^{-5} & |\xi_{\mu\mu}| < 1.0 \cdot 10^{-2} & |\xi_{\mu\tau}| < 1.0 \cdot 10^{-2} \\ |\xi_{\tau e}| < 1.6 \cdot 10^{-2} & |\xi_{\tau\mu}| < 1.0 \cdot 10^{-2} & |\xi_{\tau\tau}| < 1.0 \cdot 10^{-2} \end{pmatrix} \quad (30)$$

at the 90% confidence level. It is clear that the effects of unitarity violation can saturate the experimental upper bounds in our Type-II seesaw scenarios, only if $m_D/m_R \sim 0.1$ is taken. The latter may lead to appreciable collider signatures of lepton number violation induced by the heavy Majorana neutrinos and doubly-charged scalars.

A direct test of the seesaw mechanism requires the unambiguous observation of heavy Majorana neutrinos. The clearest signature induced by N_i should be the lepton-number-violating process $pp \rightarrow W^\pm \rightarrow \mu^\pm N \rightarrow \mu^\pm \mu^\pm jj$ at the LHC [13, 26]. For the doubly-charged scalars existing in the Type-II seesaw model, one may concentrate on either the single production $pp \rightarrow W^\pm W^\pm \rightarrow \Delta^{\pm\pm}$ [27] or the pair production in the Drell-Yan process $q\bar{q} \rightarrow \gamma^*/Z^* \rightarrow \Delta^{\pm\pm} \Delta^{\mp\mp}$ [28] and the subsequent decays $\Delta^{\pm\pm} \rightarrow W^\pm W^\pm$ or $\Delta^{\pm\pm} \rightarrow l^\pm l^\pm$. Some remarks are in order:

- The lepton-number-violating processes include both $pp \rightarrow W^\pm W^\pm \rightarrow \mu^\pm \mu^\pm jj$ and $pp \rightarrow W^\pm \rightarrow \mu^\pm N \rightarrow \mu^\pm \mu^\pm jj$ modes. The latter can be resonantly enhanced due to the on-shell production of heavy Majorana neutrinos. Given $M_i \sim 100$ GeV for example, one may follow the analysis of Ref. [13] to show that it is possible to probe $\xi_{\mu\mu}$ of $\mathcal{O}(10^{-4})$ at the 2σ level by means of the LHC with an integrated luminosity 100 fb^{-1} . Even though the background might be more complicated than naively expected [26], we feel that the discovery of heavy Majorana neutrinos with $M_i \sim \mathcal{O}(10^2)$ GeV to $\mathcal{O}(1)$ TeV and $\xi_{\mu\mu} \sim \mathcal{O}(10^{-3})$ to $\mathcal{O}(10^{-2})$ is still possible.
- Because light neutrino masses arise from the significant cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms in our Type-II seesaw scenarios, one can notice that m_L is much larger than m_i . Taking $m_R \sim 100$ GeV and $m_D/m_R \sim 0.1$ for example, we obtain $m_L \approx m_D^2/m_R \sim 1$ GeV as a consequence of cancellation. The implication of $m_L = y_\Delta \langle \Delta \rangle \sim 1$ GeV is rather clear: even if the vev of the Higgs triplet reaches the experimental upper bound $\langle \Delta \rangle \lesssim 1$ GeV, one can get a large Yukawa coupling $y_\Delta \sim \mathcal{O}(1)$. The single production rate of $W^\pm W^\pm \rightarrow \Delta^{\pm\pm}$ is proportional to $(\langle \Delta \rangle/v)^2 \sim 10^{-4}$, so this process is too small to be observed at the LHC. In Ref. [28], it has been advocated

that signatures of the doubly-charged scalars can be observed at the LHC via the pair production channel and the $l^\pm l^\pm$ decay mode with a branching fraction $\sim 50\%$ up to the mass range of 800 GeV to 1 TeV. This conclusion is applicable to our model, but the choice of $y_\Delta \sim \mathcal{O}(1)$ and $\langle \Delta \rangle \sim 1$ GeV will extend the above mass range for the doubly-charged scalars. As the total decay rate is enlarged, however, the $\Delta^{\pm\pm}$ particles cannot be the long-lived doubly-charged scalars which have been looked for at the Tevatron.

Of course, it is also possible to search for the lepton-number-violating signatures at the future International Linear Collider (ILC) via the processes $e^+e^- \rightarrow W^\pm/Z^* \rightarrow \nu N$ for the heavy Majorana neutrinos and $e^+e^- \rightarrow \gamma^*/Z^* \rightarrow \Delta^{\pm\pm}\Delta^{\mp\mp}$ for the doubly-charged scalars.

V. CONCLUDING REMARKS

The main concern of this work is the experimental testability of the seesaw mechanism in the era of LHC and (or) ILC. We have presented the most general proof of a no-go theorem, which forbids the tree-level generation of light Majorana neutrino masses if the condition $M_L = M_D M_R^{-1} M_D^T$ is satisfied in the Type-II seesaw model. Furthermore, we have shown that a compromise between tiny neutrino masses and appreciable collider signatures can be achieved by allowing for a significant but incomplete cancellation between M_L and $M_D M_R^{-1} M_D^T$ terms. In other words, observable effects of lepton number violation may be induced by the heavy Majorana neutrinos and doubly-charged scalars at the TeV scale because both M_L and $M_D M_R^{-1} M_D^T$ terms are not strongly suppressed, but their difference is tiny and responsible for the tiny masses of three light Majorana neutrinos. We have proposed three simple but viable Type-II seesaw scenarios, in which the $A_4 \times U(1)_X$ flavor symmetry is taken into account, to illustrate our main ideas.

It is worth highlighting that the non-unitarity of the lepton flavor mixing matrix V , which describes the strength of charged-current interactions of light Majorana neutrinos, is an intrinsic feature of the seesaw models. The CP-conserving and CP-violating effects of this unitarity violation can be measured or constrained in the future long-baseline neutrino oscillation experiments.

It is also worth remarking the interesting correlation between V and R , the 3×3 rotation matrix which characterizes the strength of charged-current interactions of heavy Majorana

neutrinos. As a result of $VV^\dagger = \mathbf{1} - RR^\dagger$ in both Type-I and Type-II seesaw models, larger magnitudes of the elements of R lead to larger deviations of V from unitarity (or vice versa). In this sense, testing the unitarity of V in neutrino oscillations and searching for heavy Majorana neutrinos at hadron or e^+e^- colliders are the two faces of one coin: they can be complementary to each other, both qualitatively and quantitatively, to understand the properties of light and heavy Majorana neutrinos.

Although the Type-II seesaw scenarios proposed in this paper are far from perfect, they may serve as a phenomenological example to illustrate possible ways for model building. But much more efforts are certainly needed to study neutrino physics at the TeV scale. For instance, one may question whether a compromise can still be achieved between tiny neutrino masses and appreciable collider signatures, when a successful realization of the TeV-scale leptogenesis is simultaneously required. We shall address ourselves to such difficult but interesting problems elsewhere.

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APPENDIX A: THE SCALAR POTENTIAL

In this appendix, we list the complete scalar potential in the type-II seesaw scenarios proposed in section III. For simplicity, only the first scenario is considered, and the other two cases can be discussed in a similar way. The $SU(2)_L \times U(1)_Y \otimes A_4$ invariant and renormalizable terms with the discrete Z_2 symmetry can in general be written as

$$\begin{aligned}
V(\Phi) = & \mu_\Phi^2 (\Phi^\dagger \Phi)_{\underline{1}} + \lambda_1^\Phi (\Phi^\dagger \Phi)_{\underline{1}} (\Phi^\dagger \Phi)_{\underline{1}} + \lambda_2^\Phi (\Phi^\dagger \Phi)_{\underline{1}'} (\Phi^\dagger \Phi)_{\underline{1}''} \\
& + \lambda_3^\Phi (\Phi^\dagger \Phi)_{\underline{3}_s} (\Phi^\dagger \Phi)_{\underline{3}_s} + \lambda_4^\Phi (\Phi^\dagger \Phi)_{\underline{3}_a} (\Phi^\dagger \Phi)_{\underline{3}_a} \\
& + i\lambda_5^\Phi (\Phi^\dagger \Phi)_{\underline{3}_s} (\Phi^\dagger \Phi)_{\underline{3}_a} ,
\end{aligned} \tag{A1}$$

$$\begin{aligned}
V(\chi) = & \mu_\chi^2 (\chi^\dagger \chi)_{\underline{1}} + \lambda_1^\chi (\chi^\dagger \chi)_{\underline{1}} (\chi^\dagger \chi)_{\underline{1}} + \lambda_2^\chi (\chi^\dagger \chi)_{\underline{1}'} (\chi^\dagger \chi)_{\underline{1}''} \\
& + \lambda_3^\chi (\chi^\dagger \chi)_{\underline{3}_s} (\chi^\dagger \chi)_{\underline{3}_s} + \lambda_4^\chi (\chi^\dagger \chi)_{\underline{3}_a} (\chi^\dagger \chi)_{\underline{3}_a} \\
& + i\lambda_5^\chi (\chi^\dagger \chi)_{\underline{3}_s} (\chi^\dagger \chi)_{\underline{3}_a} ,
\end{aligned} \tag{A2}$$

$$V(\phi) = \mu_\phi^2 (\phi^\dagger \phi)_{\underline{1}} + \lambda^\phi (\phi^\dagger \phi)_{\underline{1}}^2 , \tag{A3}$$

$$V(\Delta) = \mu_\Delta^2 \text{Tr} (\Delta^\dagger \Delta)_{\underline{1}} + \lambda_1^\Delta \text{Tr} (\Delta^\dagger \Delta)_{\underline{1}} \text{Tr} (\Delta^\dagger \Delta)_{\underline{1}} \\ + \lambda_2^\Delta \text{Tr} \left[(\Delta^\dagger \Delta)_{\underline{1}} (\Delta^\dagger \Delta)_{\underline{1}} \right], \quad (\text{A4})$$

$$V(\Sigma) = \mu_\Sigma^2 \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}} + \lambda_1^\Sigma \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}} + \lambda_2^\Sigma \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}'} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}''} \\ + \lambda_3^\Sigma \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{3}_s} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{3}_s} + \lambda_4^\Sigma \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{3}_a} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{3}_a} \\ + i\lambda_5^\Sigma \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{3}_s} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{3}_a} + \lambda_6^\Sigma \text{Tr} \left[(\Sigma^\dagger \Sigma)_{\underline{1}} (\Sigma^\dagger \Sigma)_{\underline{1}} \right] \\ + \lambda_7^\Sigma \text{Tr} \left[(\Sigma^\dagger \Sigma)_{\underline{1}'} (\Sigma^\dagger \Sigma)_{\underline{1}''} \right] + \lambda_8^\Sigma \text{Tr} \left[(\Sigma^\dagger \Sigma)_{\underline{3}_s} (\Sigma^\dagger \Sigma)_{\underline{3}_s} \right] \\ + \lambda_9^\Sigma \text{Tr} \left[(\Sigma^\dagger \Sigma)_{\underline{3}_a} (\Sigma^\dagger \Sigma)_{\underline{3}_a} \right] + i\lambda_{10}^\Sigma \text{Tr} \left[(\Sigma^\dagger \Sigma)_{\underline{3}_s} (\Sigma^\dagger \Sigma)_{\underline{3}_a} \right], \quad (\text{A5})$$

$$V(\Phi, \chi) = \lambda_1^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{1}} (\chi^\dagger \chi)_{\underline{1}} + \lambda_2^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{1}'} (\chi^\dagger \chi)_{\underline{1}''} + \lambda_3^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{1}''} (\chi^\dagger \chi)_{\underline{1}'} \\ + \lambda_4^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{3}_s} (\chi^\dagger \chi)_{\underline{3}_s} + \lambda_5^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{3}_a} (\chi^\dagger \chi)_{\underline{3}_a} \\ + i\lambda_6^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{3}_s} (\chi^\dagger \chi)_{\underline{3}_a} + i\lambda_7^{\Phi\chi} (\Phi^\dagger \Phi)_{\underline{3}_a} (\chi^\dagger \chi)_{\underline{3}_s}, \quad (\text{A6})$$

$$V(\Phi, \phi) = \left[\lambda_1^{\Phi\phi} (\Phi^\dagger \phi)_{\underline{3}} (\Phi^\dagger \phi)_{\underline{3}} + \text{h.c.} \right] + \lambda_2^{\Phi\phi} (\Phi^\dagger \phi)_{\underline{3}} (\phi^\dagger \Phi)_{\underline{3}} \\ + \lambda_3^{\Phi\phi} (\Phi^\dagger \Phi)_{\underline{1}} (\phi^\dagger \phi)_{\underline{1}}, \quad (\text{A7})$$

$$V(\Phi, \Delta) = \lambda_1^{\Phi\Delta} (\Phi^\dagger \Phi)_{\underline{1}} \text{Tr} (\Delta^\dagger \Delta)_{\underline{1}} + \lambda_2^{\Phi\Delta} \Phi_{\underline{3}}^\dagger [\Delta, \Delta^\dagger]_{\underline{1}} \Phi_{\underline{3}}, \quad (\text{A8})$$

$$V(\Phi, \Sigma) = \lambda_1^{\Phi\Sigma} (\Phi^\dagger \Phi)_{\underline{1}} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}} + \lambda_2^{\Phi\Sigma} \Phi^\dagger [\Sigma, \Sigma^\dagger] \Phi + [\lambda_3^{\Phi\Sigma} (\Phi^T \Phi) \Sigma + \text{h.c.}], \quad (\text{A9})$$

$$V(\chi, \phi) = \lambda_1^{\chi\phi} (\chi^\dagger \chi)_{\underline{1}} (\phi^\dagger \phi)_{\underline{1}}, \quad (\text{A10})$$

$$V(\chi, \Delta) = \lambda_1^{\chi\Delta} (\chi^\dagger \chi)_{\underline{1}} \text{Tr} (\Delta^\dagger \Delta)_{\underline{1}}, \quad (\text{A11})$$

$$V(\chi, \Sigma) = \lambda_1^{\chi\Sigma} (\chi^\dagger \chi)_{\underline{1}} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}}, \quad (\text{A12})$$

$$V(\phi, \Delta) = \lambda_1^{\phi\Delta} \phi^\dagger \phi \text{Tr} (\Delta^\dagger \Delta) + \lambda_2^{\phi\Delta} \phi^\dagger [\Delta, \Delta^\dagger] \phi + [\lambda_3^{\phi\Delta} \phi^T \phi \Delta^\dagger + \text{h.c.}], \quad (\text{A13})$$

$$V(\phi, \Sigma) = \lambda_1^{\phi\Sigma} \phi^\dagger \phi \text{Tr} (\Sigma^\dagger \Sigma) + \lambda_2^{\phi\Sigma} \phi^\dagger [\Sigma, \Sigma^\dagger] \phi, \quad (\text{A14})$$

$$V(\Delta, \Sigma) = \lambda_1^{\Delta\Sigma} \text{Tr} (\Delta^\dagger \Delta)_{\underline{1}} \text{Tr} (\Sigma^\dagger \Sigma)_{\underline{1}} + \lambda_2^{\Delta\Sigma} \text{Tr} \left[(\Delta^\dagger \Delta)_{\underline{1}} (\Sigma^\dagger \Sigma)_{\underline{1}} \right] \\ + \lambda_3^{\Delta\Sigma} \text{Tr} \left[(\Delta^\dagger \Sigma)_{\underline{3}} (\Delta \Sigma^\dagger)_{\underline{3}} \right], \quad (\text{A15})$$

$$V(\Phi, \phi, \chi) = \lambda^{\Phi\phi\chi} (\phi^\dagger \Phi)_{\underline{3}} \chi_{\underline{3}} + \text{h.c.} \quad (\text{A16})$$

Note that the above scalar potential also respects the $U(1)_X$ symmetry except for the terms of $V(\Phi, \phi)$ in the square bracket in Eq. (A7), which explicitly breaks $U(1)_X$ to Z_2 .

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